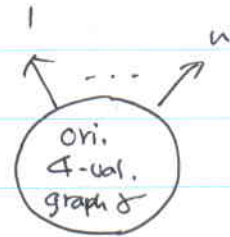


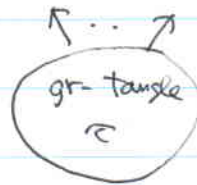
Rozansky

joint with M. Khovanov

$$SU(N) \rightarrow W(x) = x^{N+1} \\ \uparrow \\ \mathbb{R}[x]$$



$$\mapsto \hat{\delta} \in MF_{\sum_{i=1}^n W(\delta_i)}$$



$$\mapsto \hat{c} \in Kom(MF_{\sum W(\delta_i)})$$

$$K(p, g) := (R_1 \xrightleftharpoons[p]{g} R_0) \quad , \quad K(p, c) = \bigotimes_{c_i}^n K(p, g_i)$$

$$\xrightarrow[1]{2} = K(x_2 - x_1, W(x_1, x_2))$$

$$\begin{matrix} 3 \\ \uparrow \\ 1 \end{matrix} \begin{matrix} 4 \\ \uparrow \\ 2 \end{matrix} = K \left( \begin{matrix} x_3 - x_1 : W(x_1, x_3) \\ x_4 - x_2 : W(x_2, x_4) \end{matrix} \right) \cong K \left( \begin{matrix} x_3 + x_4 - x_1 - x_2 : a \\ x_4 - x_2 : (x_4 - x_1) b \end{matrix} \right)$$

$$\begin{matrix} \nearrow \\ \searrow \end{matrix} = K \left( \begin{matrix} x_3 + x_4 - x_1 - x_2 : a \\ (x_4 - x_1)(x_4 - x_2) : b \end{matrix} \right)$$

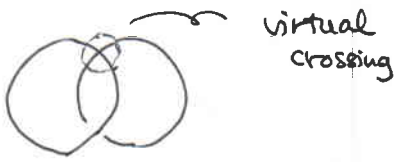
$\begin{matrix} 3 \\ \uparrow \\ 1 \end{matrix} \begin{matrix} 4 \\ \uparrow \\ 2 \end{matrix}$

$\begin{matrix} R_1 & \xrightarrow{x_4 - x_2} & R_0 & \xrightarrow{(x_4 - x_1)b} & R_1 \\ \downarrow & & \downarrow & & \downarrow \\ R_1 & \xrightarrow{x_4 - x_1} & R_0 & \xrightarrow{(x_4 - x_2)b} & R_1 \end{matrix}$

$$\begin{matrix} \nearrow \\ \searrow \end{matrix} = \left( \begin{matrix} \nearrow \\ \searrow \end{matrix} \xrightarrow{x_{in}} \begin{matrix} \nearrow \\ \searrow \end{matrix} \right) \langle 1 \rangle \quad \begin{matrix} \nearrow \\ \searrow \end{matrix} = \left( \begin{matrix} \nearrow \\ \searrow \end{matrix} \xrightarrow{x_{out}} \begin{matrix} \nearrow \\ \searrow \end{matrix} \right) \langle 1 \rangle$$

$\begin{matrix} \nearrow \\ \searrow \end{matrix}$	$R_1 \xrightarrow{(x_4 - x_2)} R_0 \xrightarrow{(x_4 - x_1)b} R_1$	
$\downarrow x_{in}$	$\downarrow (x_4 - x_2) \quad \downarrow (x_4 - x_1)$	
$\begin{matrix} \nearrow \\ \searrow \end{matrix}$	$R_1 \xrightarrow{(x_4 - x_1)} R_0 \xrightarrow{b} R_1$	$W = p \otimes r$
$\downarrow x_{out}$	$(x_4 - x_1) \downarrow \quad \downarrow 1 \quad \downarrow (x_4 - x_1)$	$(p \otimes r) \downarrow$
$\left( \begin{matrix} \nearrow \\ \searrow \end{matrix} \right)$	$R_1 \xrightarrow{x_4 - x_2} R_0 \xrightarrow{(x_4 - x_1)b} R_1$	$p \otimes (r)$

Kauffman invented a notion of a virtual link



$$R1' \quad \text{diagram} \sim ($$

$$R2' \quad \text{diagram} \sim )($$

$$R3' \quad \text{diagram} \sim \text{diagram}$$

$$\text{diagram} \sim \text{diagram}$$

$$\text{diagram} \neq \text{diagram}$$

{ links }  $\dashrightarrow$  { virt. links }

$$\text{X} \rightarrow \cup, \text{)(}$$

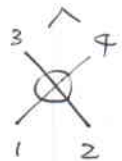
SU(N) does not quite work



T

$$V \otimes W \rightarrow W \otimes V$$

matrix  
trac.

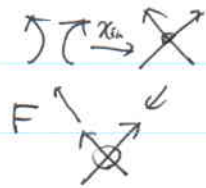


$$= K \begin{pmatrix} x_3 - x_2 & W(x_2, x_3) \\ x_4 - x_1 & W(x_1, x_4) \end{pmatrix}$$

$$\cong K \begin{pmatrix} - & - \\ x_4 - x_1 & (x_4 - x_2)b \end{pmatrix}$$

$$\text{Cone}_{MF} \left( \begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array} \xrightarrow{\chi_{in}} \begin{array}{c} \times \\ \times \end{array} \right) = \begin{array}{c} \times \\ \times \end{array}$$

dist. triangle



$$\begin{array}{c} \times \\ \times \end{array} = \boxed{\begin{array}{c} \times \\ \times \end{array} \xrightarrow{F} \begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array}} \quad \text{or cone of } F$$

similarly

$$= \boxed{\begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array} \xrightarrow{G} \begin{array}{c} \times \\ \times \end{array}}$$

$$\begin{array}{c} \times \\ \times \end{array} = \begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array} \xrightarrow{\begin{pmatrix} 0 \\ 1 \end{pmatrix}} \boxed{\begin{array}{c} \times \\ \times \end{array} \xrightarrow{F} \begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array}}$$

$$\begin{array}{c} \times \\ \times \end{array} = \boxed{\begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array} \xrightarrow{G} \begin{array}{c} \times \\ \times \end{array}} \xrightarrow{\begin{pmatrix} 1 \\ 0 \end{pmatrix}} \begin{array}{c} \hookrightarrow \\ \hookrightarrow \end{array}$$

"homological" deformation of an object by a morphism

$$\mathcal{C} : \text{triangulated} \quad A, B \in \mathcal{C} \quad A[1] \xrightarrow{F} B$$

$$A_F, B_F \in \text{Kam}(\mathcal{C})$$

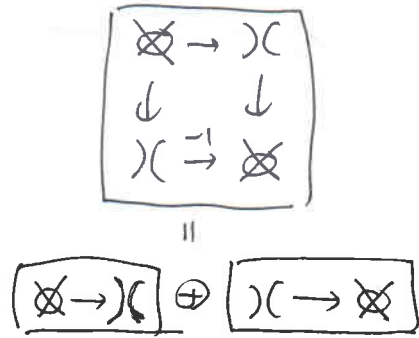
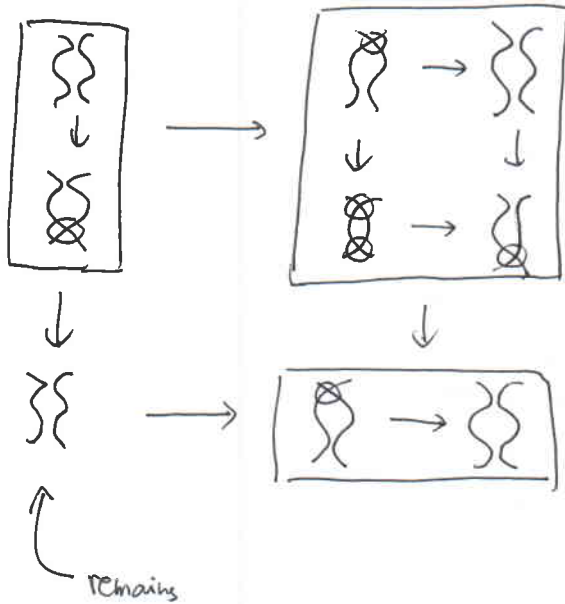
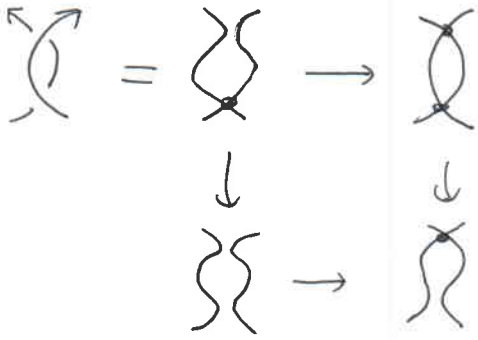
$$\begin{array}{ccc} A[1] & \xrightarrow{F} & B \\ \alpha \swarrow & & \swarrow \beta \\ & \text{Cone}(F) & \end{array}$$

$$A_F = \text{Cone}_{\text{Kam}(\mathcal{C})}(\beta) = B \rightarrow \boxed{A \rightarrow B}$$

$$B_F = \text{Cone}_{\text{Kam}(\mathcal{C})}(\alpha) = \boxed{A \rightarrow B} \rightarrow A$$

$$\therefore \begin{array}{c} \times \\ \times \end{array} = \begin{array}{c} \times \\ \times \end{array}_F \quad \begin{array}{c} \times \\ \times \end{array} = \begin{array}{c} \times \\ \times \end{array}_G$$

standard one



NB  
no ext between

R2

Kauffman pol.  
SO(2N+2)

$$\times - \times' = (q - q^{-1}) ( ) ( - \cup )$$

$$\cup = q^{2N+1} \quad | \quad (\text{but not important})$$

$$\text{Link not} = \frac{q^{2N+1} - q^{-(2N+1)}}{q - q^{-1}} + 1$$

diagram  $L \mapsto C(L)$  of  $\mathbb{Z} \times \mathbb{Z}_2$ -graded vector sp.

$$C(\cup) \simeq C(|) \{-2N-1\} \langle 1 \rangle [1]$$

$$\deg_{\mathbb{Z}_2}(C(L)) = n_L \pmod{2} \text{ where } n_L = \# \text{ of crossing in } L$$

$$\begin{aligned} X &= -X + \delta \left( + \delta^{-1} \cup \right) \\ &= -X + \delta \left( + \delta^{-1} \cap \right) \end{aligned}$$

Gutov-Walden

$$W(x, y) = x^{2N+1} + xy^2$$

$$\frac{1}{2} = K \begin{pmatrix} x_2 + x_1 & ? \\ y_2 + y_1 & ? \end{pmatrix}$$

$$i \cap j \quad / \begin{pmatrix} x_i + x_j \\ y_i + y_j \end{pmatrix}$$

$$\bigcirc = \frac{1}{2} / (x_1 + x_2, y_1 + y_2)$$

$$\deg_x x = 2$$

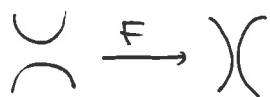
$$\deg_x y = 2N$$

$\rightsquigarrow$  Pointnet

Big Quest.  $\hat{X} = ?$

Use virtual crossing trick!

Find saddle mod



F lowest  $\mathbb{Z}_2$ -degree in  $\text{Exp}_0(\cup, \cup)$



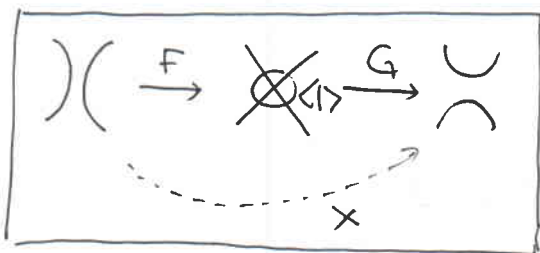
$$G \circ F \approx 0$$

$$\Rightarrow \exists X \in \text{Hom}_{\mathbb{Z}_2}(\cup, \cup)$$

$$\deg_{\mathbb{Z}_2} X = 1$$

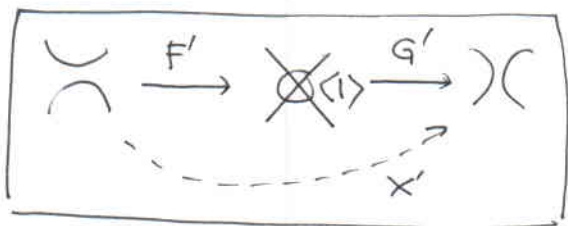
convolution

$$\text{st. } GF = -\{D, X\}$$



$$=: X$$

~~MF~~



i.e. rotationally inv.

(discard  $\mathbb{Z}_2$ -mult)

$$X := (\cup \rightarrow \boxed{\cup \xrightarrow{F} \text{crossed} \xrightarrow{G} \cup}) \rightarrow \cup$$

(R1), (R2) OK

(R3) within the reach