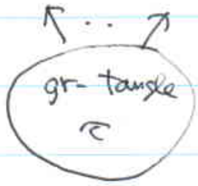
Rozansky

joint with M. Khovanov

$SU(N) \to W(x) = x^{N+1}$

$\qquad \qquad \mathbb{Q}[x]$

$\mapsto \hat{\gamma} \in MF_{\sum_{i=1}^{n} W(x_i)}$



$\mapsto \hat{\tau} \in Kom(MF_{\sum W(x_i)})$

$K(p,q) := (R_1 \overset{p}{\underset{q}{\rightleftarrows}} R_0)$, $K(\vec{p},\vec{q}) = \bigotimes_{i=1}^{n} K(p_i, q_i)$

$\widehat{\underset{1 \quad 2}{\longrightarrow}} = K(x_2 - x_1, W(x_1, x_2))$

$\widehat{\text{[pair of arcs } 1\to3,\ 2\to4]} = K\begin{pmatrix} x_3 - x_1 : W(x_1,x_3) \\ x_4 - x_2 ; W(x_2,x_4) \end{pmatrix} \cong K\begin{pmatrix} x_3+x_4-x_1-x_2 : a \\ x_4 - x_2 : (x_4 - x_1)b \end{pmatrix}$

$\widehat{\text{[crossing]}} = K\begin{pmatrix} x_3 + x_4 - x_1 - x_2 & : a \\ (x_4 - x_1)(x_4 - x_2) & : b \end{pmatrix}$

[pair of arcs] $\downarrow F$ [crossing]: $R_1 \xrightarrow{x_4 - x_2} R_0 \xrightarrow{(x_4-x_1)b} R_1$, $\downarrow 1$, $\downarrow 1$, $R_1 \xrightarrow[x_4 - x_1]{} R_0 \xrightarrow[(x_4-x_2)b]{} R_1$

$\widehat{\text{[crossing]}} = \left(\text{[pair of arcs]} \xrightarrow{x_{in}} \text{[crossing]} \right)\langle 1 \rangle$ $\qquad$ $\widehat{\text{[crossing]}} = \left(\text{[crossing]} \xrightarrow{x_{out}} \text{[pair of arcs]} \right)\langle 1 \rangle$

[pair of arcs] $\qquad R_1 \xrightarrow{(x_4 - x_2)} R_0 \xrightarrow{(x_4 - x_1)b} R_1$

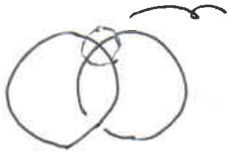
$\downarrow x_{in} \qquad 1 \downarrow \qquad \downarrow x_4 - x_1 \qquad \downarrow 1$

[crossing] $\qquad R_1 \xrightarrow[(x_4 - x_1)]{(x_4 - x_2)} R_0 \xrightarrow{b} R_1$ $\qquad W = pqr$

$\downarrow x_{out} \qquad (x_4 - x_1)\downarrow \qquad \downarrow 1 \qquad \downarrow x_4 - x_1$ $\qquad (pq)r$

)($\qquad R_1 \xrightarrow{x_4 - x_2} R_0 \xrightarrow{(x_4 - x_1)b} R_1$ $\qquad p(qr)$

Kauffman invented a notion of a virtual link



R1′ 

R2′ 

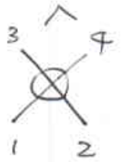
R3′ 

} links{ --> { virt. links}

SU(N) does not quite work

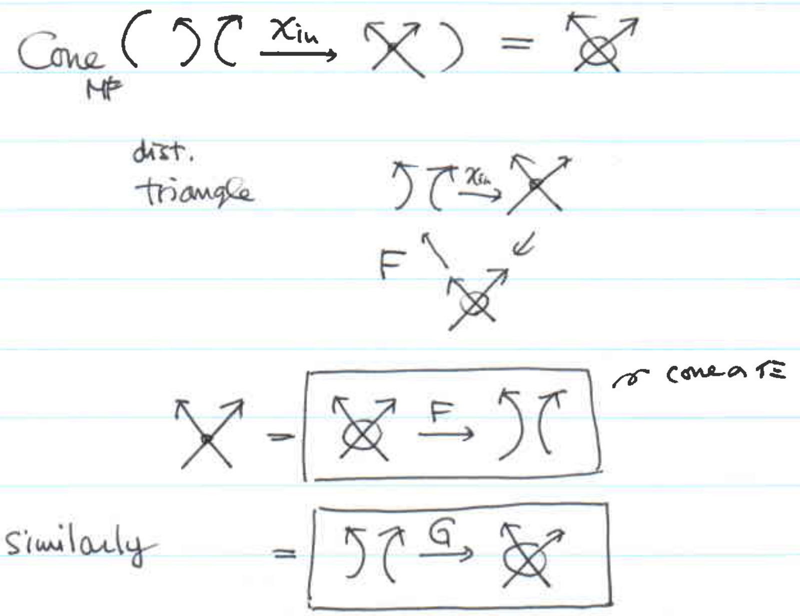
$V \otimes W \to W \otimes V$

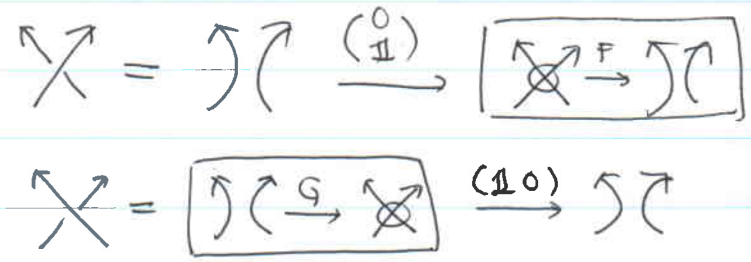
matrix fac.



$$= K\begin{pmatrix} x_3 - x_2 & W(x_2, x_3) \\ x_4 - x_1 , & W(x_1, x_4) \end{pmatrix}$$

$$\cong K\begin{pmatrix} \text{—//—} \\ x_4 - x_1 , & (x_4 - x_2)b \end{pmatrix}$$

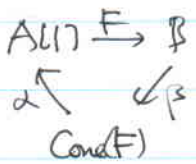




"homological" deformation of an object by a morphism

$\mathcal{C}$: triangulated $\quad A, B \in \mathcal{C} \quad A[1] \xrightarrow{F} B$

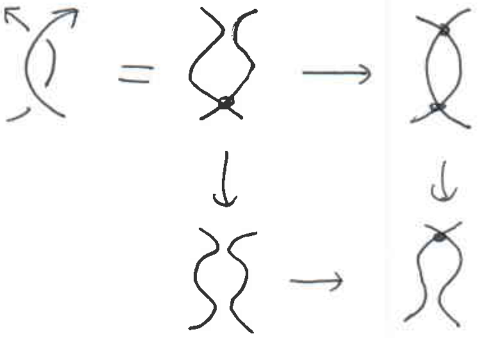
$A_F, B_F \in Kom(\mathcal{C})$

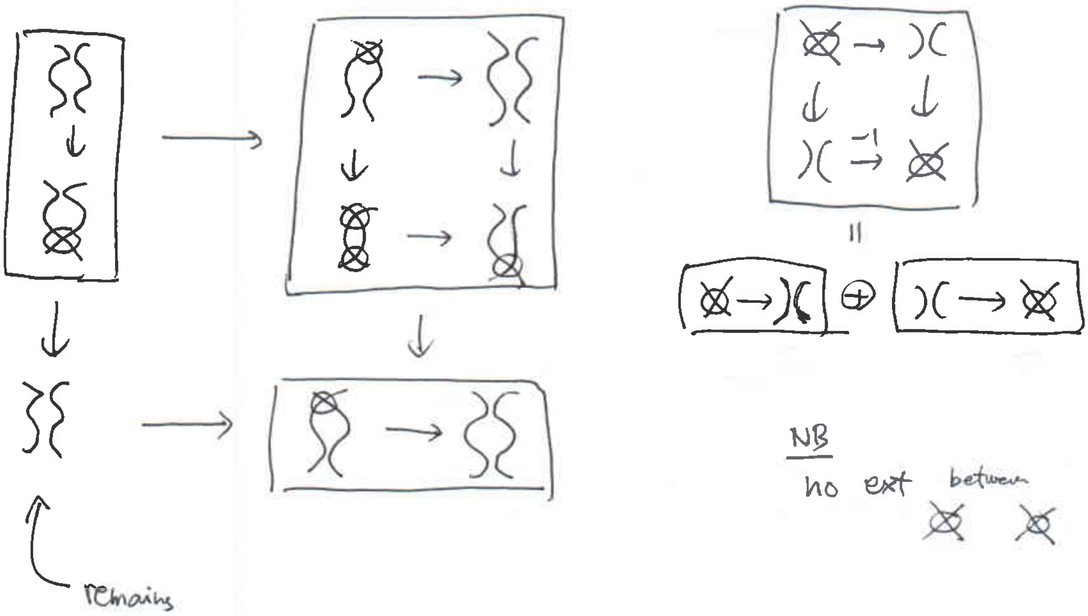


$$A_F = \mathrm{Cone}_{Kom(\mathcal{C})}(\beta) = B \to \boxed{A \to B}$$

$$B_F = \mathrm{Cone}_{Kom(\mathcal{C})}(\alpha) = \boxed{A \to B} \to A$$

stdard one

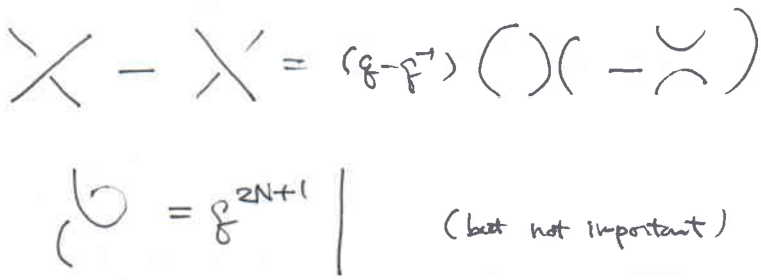




R2

—— . ——

Kauffman pol.
SO(2N+2)

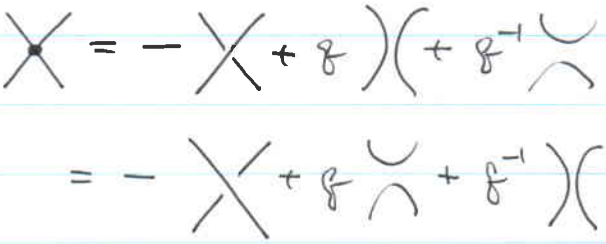


(but not important)

$$P_{unknot} = \frac{q^{2N+1} - q^{-(2N+1)}}{q - q^{-1}} + 1$$

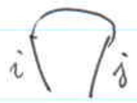
diagram $L \longmapsto C^\cdot(L)$ of $\mathbb{Z}\times\mathbb{Z}_2$-graded vector sp.

$\deg_{\mathbb{Z}_2}(C^\cdot(L)) = n_L \pmod 2$ where $n_L = \#$ of crossing in L



Gukov-Walcher $\quad W(x,y) = x^{2N+1} + xy^2$

 $= K\begin{pmatrix} x_2+x_1 & ; & ? \\ y_2+y_1 & ; & ? \end{pmatrix}$

 $\Big/\begin{pmatrix} x_i+x_j \\ y_i+y_j \end{pmatrix}$

 $=$  $\Big/ (x_1+x_2,\ y_1+y_2)$

$\deg_q x = 2$

$\deg_q y = 2N$

$\rightsquigarrow$ Punknot

Big Quest.  $= ?$ Use virtual crossing trick!

Find saddle mor

F lowest q-degree in $\mathrm{Ext}_0(\asymp,)()$

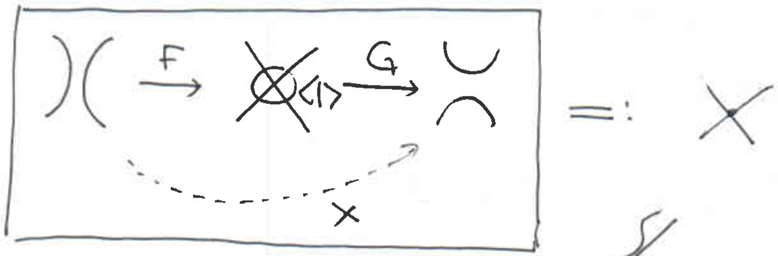
$G \circ F \simeq 0$

$\Rightarrow \exists X \in \mathrm{Hom}_R()(, \asymp)$

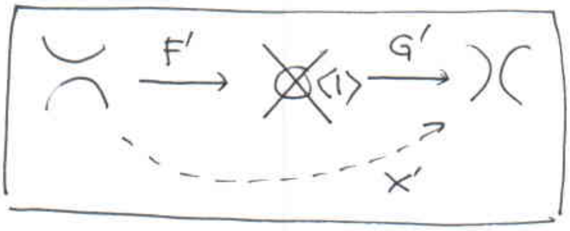
$\deg_{\mathbb{Z}_2} X = 1$

s.t. $GF = -\{D, X\}$

convolution

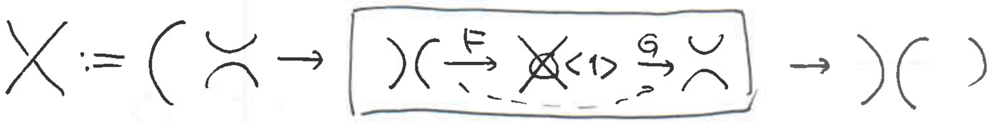


$\simeq_{MF}$



i.e. rotationally inv.

(discard q-shift



(R1), (R2) OK

(R3) within the reach